Solving Shortest Path Problems Using Mathematical Literacy Skill Figured Out By Pre-Service Teachers

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Solving Shortest Path Problems Using Mathematical Literacy Skill Figured Out By Pre-Service Teachers

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Abstract. Mathematical literacy is an important ability to be taught to the mathematics pre-service teacher. The students who have good mathematical literacy skills will be sensitive to the mathematical concepts which are relevant to the phenomenon or problem. One of the problems that is given to mathematics pre-service teacher is the shortest path problem in Graph Theory courses. The purpose of this study is to explore the mathematical literacy abilities of the pre-service teacher in solving the shortest path problems. The research subjects were 56 mathematics pre-service teacher of Universitas PGRI Madiun, Indonesia. The data were collected through problem-solving tests which were then analyzed based on indicators of mathematical literacy ability namely formulate, employ, and interpret. The results shows that the mathematical literacy abilities of pre-service teacher in solving the shortest path problem consist of three categories, namely complete, incomplete, and incomplete mathematical literacy abilities. There are 9 students with complete mathematical literacy skills can formulate, employ, and interpret. There are 18 students with incomplete mathematical literacy skills can formulate and employ. There are 29 students with incomplete mathematical literacy can only formulate.

1. Introduction

The development of science and technology raise an obligation for the students to master mathematics. Mathematics is a universal science that underlies and deals with science and technology that has functions in various other multi-disciplines. Various life problems can be solved by mathematical thinking. Moreover, it teaches someone who learns it to be able to think logically, critically, analitically, systematically, and creatively. This is one of the reasons, why it is always studied at every level of formal education [1].

Recently, mathematics is still considered as a difficult material to learn in Indonesia. This is in line with the results of the PISA in 2015. The average mathematical score of Indonesian students is 386. It is still far from the average international score of 386. These results place Indonesia in the rank of 62 out of 70 involved participating of PISA [2]. While the results of a survey by the International Association for the Evaluation of Educational Achievement (IEA) that measures the development of mathematics and natural sciences in the 2015 TIMSS places Indonesia in position 45 out of 48 in the field of Natural Sciences and position 45 of 50 countries in mathematics [3]. From the results of PISA 2015 and TIMMS 2015, Indonesian students' mathematical literacy is still very low in problem solving that requires the ability to examine, give reason, and communicate effectively. Furthermore, it is needed to solve and interpret the problems in a variety of contexts. Indonesia always ranks among in

the lowest ranks and also the scores obtained are always below average, which obviously is not the expected result [4]. In his research, Wijaya [5] shows that students do not reach the three components of information literacy; namely recognizing information needs, finding and evaluating the quality of information, and making information effectively. This shows that students' information literacy is still low.

The OECD/PISA mathematical literacy domain is concerned with the capacities of students to analyze, give reason, and communicate ideas effectively as they pose, formulate, solve and interpret mathematics in a variety of situations. The assessment focuses on real-world problems[6]. Meanwhile, some researches that have been done in several Indonesian schools show that the students are still not familiar with the problems that require logical thinking and applicative. This is one of the causes of mathematics considered as the difficulty by students because students are still fond of and accustomed to theoretical, and procedural answers. Giani, Zulkardi & Hiltrimartin [7] find no problems at the cognitive level C5 (evaluation) and C6 (create) on the Competency test questions on the BSE Mathematics Curriculum of KTSP on the subject of equality and one variable linear inequality. The absence of questions at levels C5 and C6 are because the teachers' assessment only emphasizes the recognition or recall of facts so that students are accustomed to have the knowledge at this level only.

Learning mathematics does not only require memorization and procedural skills, but also it has mathematical literacy skills. Mathematics plays a dominant role in today's world. Although not everyone will become a mathematical expert, from an educational point of view, it is the key for everyone to acquire a certain level of mathematical literacy, which allows reflecting and assessing an important mathematical processes in every day live [8], [9]. Mathematics literacy does not imply the detailed knowledge of calculus, differential equations, topology, analysis, linear algebra, abstract algebra, and complex of mathematical formulas, but rather a broad understanding and appreciation of what mathematics is capable of achieving [10], [11]. Moreover, Stacey [12] states mathematical literacy is intended to highlight mathematical skills and understand the of use of future life. Then, the intention does not refer only to simple mathematics which involved in straightforward activities such as shopping, it also encompasses preparation to use mathematics in technical professions of the highest level.

Mathematical literacy is about applying mathematics to 'real' problems [6], [10], [13], [14]. It means that these problems are not 'purely' mathematical but are placed in some kinds of a 'situation'. In short, the students have to 'solve' a real world problem which requires to use the skills and competencies they have acquired through schooling and life experiences [6]. In mathematics literacy, the students must be able to link several concepts in mathematics. This process is related to formulate the real-world problems in mathematical terms so that the students can solve the mathematical problems, and then these mathematical solutions can be interpreted to provide answers to real-world problems [12]. Mathematical literacy is an individuals' capacity to formulate, employ, and interpret mathematics in a variety of contexts [13]. These three words, "formulate", "employ" and "interpret", provide a useful and meaningful structure for organizing the mathematical processes that describe what individuals do to connect the context of a problem with the mathematics and thus to solve the it. The OECD illustrates the literacy mathematical model as in Figure 1 below.

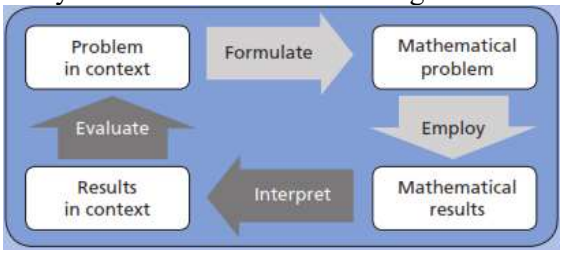


Figure 1. Mathematical Literacy Model [13]

The model is described as follows [13]. Formulation in the mathematical literacy refers to the ability of a person to recognize and identify in using and then provides a mathematical structure on contextual problems. Employing (applying) in the mathematical literacy means able to apply

mathematical concepts, facts, procedures, and mathematical reasoning to solve mathematical problems so as to obtain mathematical conclusions. Interpret in mathematical literacy focuses on the ability of individuals to reflect mathematical solutions, the results are obtained or concluded and interpreted in the real context of everyday life.

Thus, mathematical literacy ability is an important to be possessed by mathematics education students. They are prepared for improving students in facing the mathematics problem to enhance mathematical literacy skills at school. Literacy has an influence on students' understanding of the problem in learning [15]. Good mathematical skills are expected to help to maximize the task and the role of pre-service teacher in their assignments. Mathematical literacy focuses on the ability of students to analyze, justify, and communicate ideas effectively, formulate, solve and interpret mathematical problems in various forms and situations. So, pre-service teacher must have the ability to explain problem solving [16] and to teach mathematical literacy to students.

One of the courses that can be used to explore the ability of mathematical literacy through problem solving is the Graph Theory. In the Graph Theory, there are contextual material that can be used to design mathematical literacy problems, namely the shortest path. The students are required to analyze the problem then look for the shortest distance from one city to another. The cities in the context of graph theory are represented as vertices and roads connecting the two cities as edges. The students must be able to represent contextual problems in mathematical terms in this case graph form, then students can find/ calculate the shortest path using the Djikstra algorithm. Based on the description above, the purpose of this article is to explore the mathematical literacy abilities of pre-service teacher in solving the shortest path problem.

2. Research Method

This research is a descriptive study with a qualitative approach with 56 participants, pre-service teacher of mathematics department at Universitas PGRI Madiun who are currently taking a course on Graph Theory. Data collection was done through mathematical literacy ability tests on the shortest path problems as in figure 2 below. This mathematical literacy ability test has been content validated to mathematics education experts at Universitas PGRI Madiun.

An freight forwarding company has branches in 7 cities in East Java, namely (a) Madiun, (b) Ponorogo, (c) Nganjuk, (d) Kediri, (e) Blitar, (f) Malang, (g) Jombang, and (h) Stone. The distance (in km) from one city to another city is given by the element matrix A which is located on the i-th row and j-column (symbol ∞ means there is no direct path). The freight forwarding company will send an important package from the city of Jombang to Kediri. For this reason, the company needs the closest route (ignoring other factors), determine the shortest distance and the closest route from the city of Jombang to Kediri using the Dijkstra Algorithm!

$$A = \begin{matrix} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{matrix} \begin{matrix} \begin{matrix} a & b & c & d & e & f & g & h \end{matrix} \\ \begin{bmatrix} 0 & 26 & 46 & \infty & \infty & \infty & \infty & \infty \\ 26 & 0 & \infty & \infty & 90 & \infty & \infty & \infty \\ 46 & \infty & 0 & 25 & \infty & \infty & 41 & \infty \\ \infty & \infty & 25 & 0 & 30 & \infty & 42 & 50 \\ \infty & 90 & \infty & 30 & 0 & 60 & \infty & \infty \\ \infty & \infty & \infty & \infty & 60 & 0 & \infty & 15 \\ \infty & \infty & 41 & 42 & \infty & \infty & 0 & 80 \\ \infty & \infty & \infty & 50 & \infty & 15 & 80 & 0 \end{bmatrix} \end{matrix}$$

Figure 2. Mathematical Literacy Tests on Shortest Path Problems

The written test results were then analyzed based on the components of their mathematical literacy abilities. As for the indicators of mathematical literacy ability on the shortest path problem, it is presented in table 1 as follows.

Table 1. Indicators of Mathematical Literacy in Shortest Path Problems

Components of Mathematical Literacy	Indicators
Formulate	Students can recognize and identify using mathematical structures on the shortest path problems such as graphs
Employing	Students are able to apply mathematical concepts, facts, procedures, or mathematical reasoning such as the Djikstra algorithm to solve the shortest path problems so as to get answers mathematically
Interpret	Students can reflect mathematical solutions (results obtained) and interpret them into the real context of everyday life on the shortest path problem

The results of the analysis of mathematical literacy abilities were then grouped based on the completeness of the mathematical literacy components and one participant was chosen to represent each group. Each participant was chosen based on their communication skills to describe their mathematical literacy abilities based on written tests and interviews. To ensure the credibility of the data in this study triangulation of methods, tests and interviews were used.

3. Result and Discussion

From 56 students who are mathematics pre-service teachers who do mathematical literacy tests on the shortest path problem, there are 3 mathematical literacy groups, namely complete mathematical literacy that satisfies 3 components, incomplete mathematical literacy that satisfies 2 components, and incomplete mathematical literacy that only satisfies 1 component.

Based on the result, there are 9 mathematics pre-service teachers satisfies 3 components of mathematical literacy. They are able to identify relevant and available information. Moreover, the students also can describe it in the form of weighted graphs. They are able to apply mathematical concepts, facts, procedures, and mathematical reasoning in this case applying the Djikstra algorithm to determine the shortest path to solve the problem. The students are able to conclude and interpret the results of the calculation in the real context according to the problem.

There are 18 pre-service teacher in incomplete groups because they only satisfies 2 mathematical literacy components. They are able to draw weighted graphs and do calculations with Djikstra algorithm correctly, but they are unable to interpret the results of the calculations according to the context in the problem.

Whereas, there are 29 pre-service teacher in the incomplete groups that only meet one component of mathematical literacy. In this group the pre-service teachers are only able to draw weighted graphs and calculations. Thus, there are even some students who cannot represent what is known in the problem to the mathematical model.

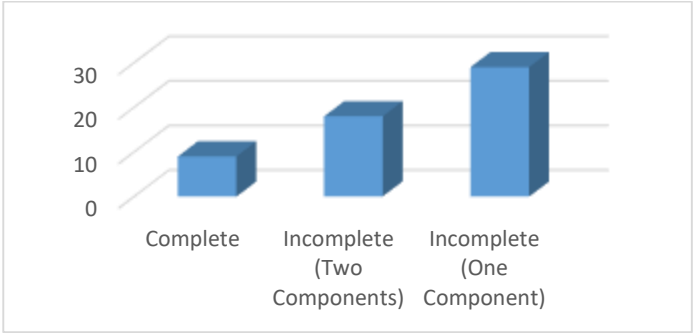


Figure 3. Mathematical Literacy Capability Diagram of Prospective Teachers in Solving Shortest Path Problems

From the three groups of mathematical literacy abilities, each participant of the mathematics pre-service teacher present to describe their mathematical literacy abilities. For the complete group, one student is chosen, namely MS. For the incomplete (two components) group, one student of mathematics pre-service teacher is also chosen, namely LR. Whereas for the incomplete (two components) group, one student is chosen, namely FO. The following is described each groups' mathematical literacy abilities.

3.1. Participant MS (Complete)

From the results of the work of participant MS, it appears that MS is able to identify what is known and is able to understand the purpose of the problem. MS is able to make a mathematical model by describing what is known to be a weighted graph as shown in figure 4 below.

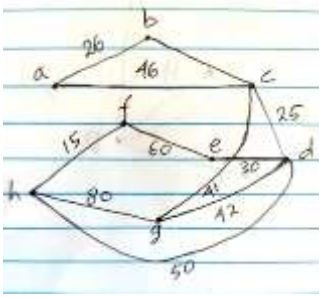
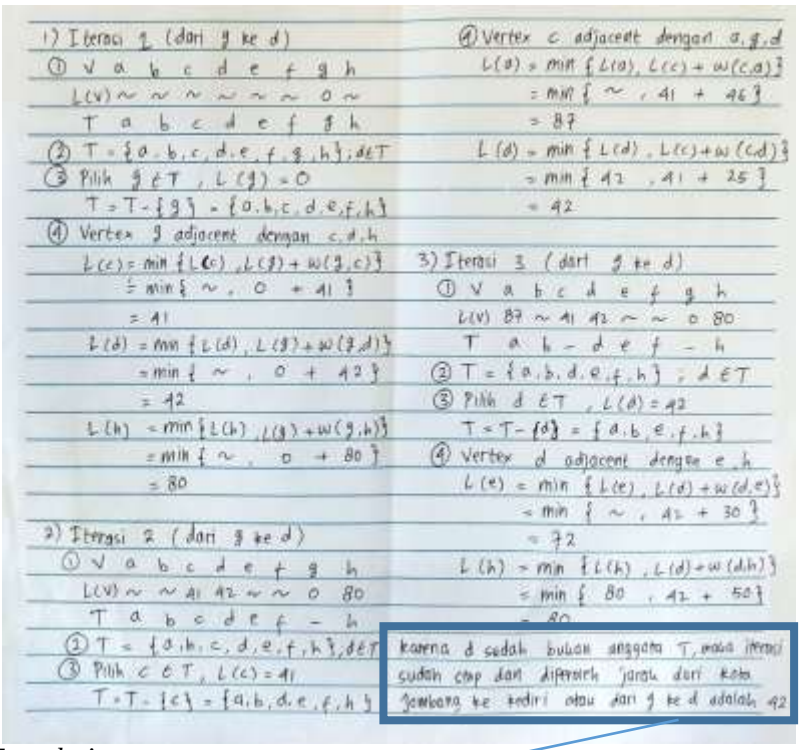


Figure 4. Weighted Graph of Participant MS

In Figure 4, it appears that MS can formulate to identify using the mathematical structure of the shortest path problem in the form of a weighted graph.



Translation:
because D is not a member of T, the iteration is stopped, so the distance from Jombang to Kediri or from g to d is 42 km

Figure 5. Application of *Dijkstra's Algorithm* and Interpretation of Results by MS

Then, figure 5 shows that MS is able to apply the Djikstra algorithm correctly to find the shortest path. This shows that MS can do better employing. In addition, MS is also able to interpret the results of the calculations according to the context on the shortest distance problem. This is also supported by MS' interview data stating that "*the results of these calculations mean that the closest distance from Jombang can go directly to Kediri without having to pass through other cities*". This shows that MS has better interpretation skills because she/ he can bring mathematical solutions directly applied to the real context [17]. As a mathematics pre-service teacher, MS has succeeded in building his idea in choosing the problem solving strategy [18].

3.2. Participant LR (Incomplete/Two Components)

LR is able to identify the information in the problem, to create a mathematical model by describing what is known to be a weighted graph as shown in Figure 6.

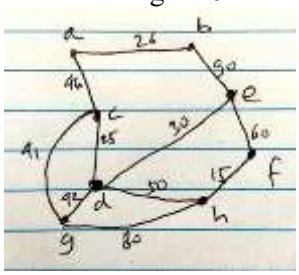


Figure 6. Weighted Graph of Participant LR

Figure 6 shows that LR can formulate, which is identifying using the mathematical structure of the shortest path problem in the form of a weighted graph.

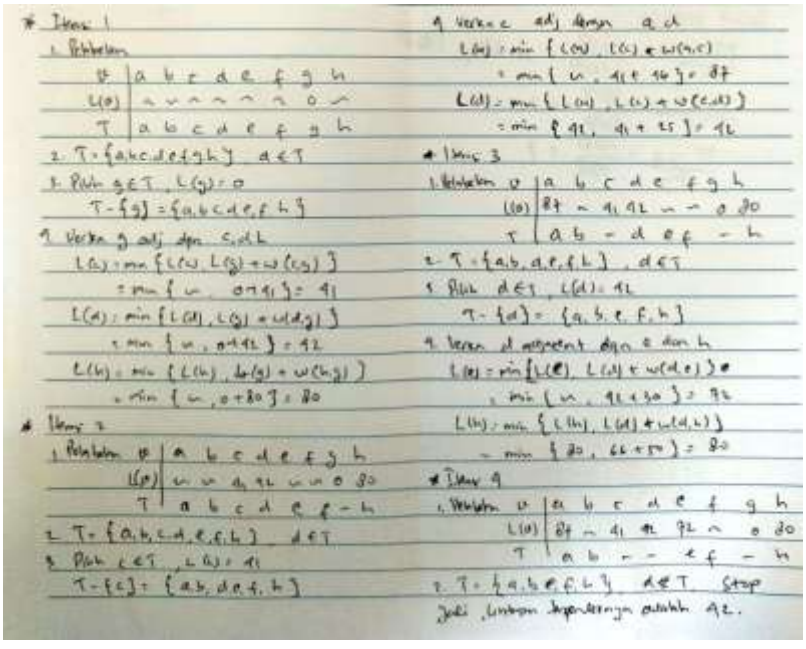


Figure 7. Application of Dijkstra's Algorithm by Participant LR

Furthermore, in figure 7, it appears that LR is also able to find the shortest run off with their mathematical concepts. LR can do employing well, she/ he is able to apply the Djikstra algorithm until a mathematical solution is obtained. However, after obtaining a mathematical solution, LR does not interpret the solutions. LR does not interpret the results of the calculations into the context of the shortest path problem. This is supported by the LR statement, "*So far I am not accustomed to return my answers to the questions. Because all this time, if there are mathematical problems, I tend to only*

do them mathematically without returning to the problem of what exactly is asked of the problem ". The third component of mathematical literacy is not carried out by LR because the habit of doing math problems so far has not been well understood. This certainly contradicts the statement that a mathematics pre-service teacher must be able to teach problems in real life well [19].

3.3. Participant FO (Incomplete/One Component)

FO is able to identify the information in the problem well by creating a mathematical model by describing what is known to be a weighted graph as shown in Figure 8. below.

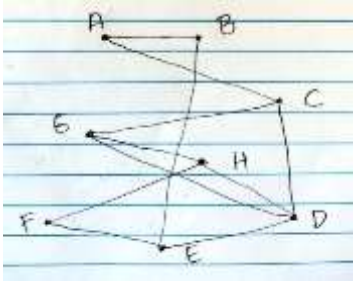


Figure 8. Weighted Graph of Participant FO

Figure 8 shows that LR can formulate to identify using the mathematical structure of the shortest path problem in the form of graph alone without giving weight to each edge.

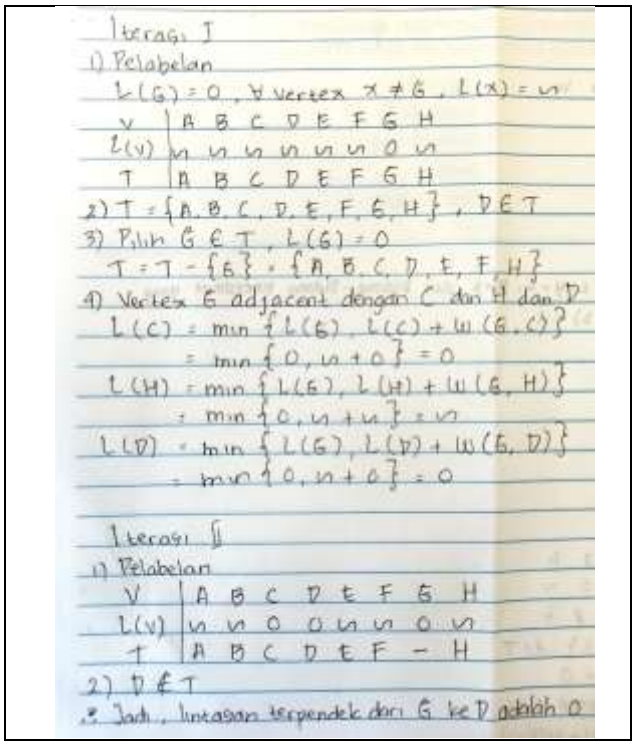


Figure 9. Application of Dijkstra's Algorithm by Partisipant FO

Furthermore, based on figure 9, it appears that FO has not been able to apply the Djikstra algorithm correctly to find the shortest path. FO has not been able to employ, because she/ he has not been able to apply the Djikstra algorithm correctly. As a result, FO also has not been able to interpret because she/ he cannot interpret the results of the calculations. This is supported by FO statement, "*I do not really master the material well, so I cannot find a solution*". As a mathematics pre-service teacher, the ability possessed by FO needs to be sought for a solution immediately, bearing in mind that to be a future professional teachers must have good content knowledge [20], [21].

4. Conclusion

Based on the results above, it can be concluded that the mathematical literacy ability of mathematics pre-service teacher at *Universitas PGRI Madiun* is diverse. From 56 students, 3 groups of mathematical literacy abilities are obtained, namely: (1) a complete group with 9 participants, (2) incomplete (two components) group with 18 participants, and (3) incomplete / one components group with 29 participants. The complete group has better mathematical literacy abilities, fulfills formulate, employ, and interpret components. As for the incomplete group, the mathematical literacy ability is not good, consisting of two groups, which only meet 2 components (formulate and employee) and only 1 meets component (formulate). It is suggested to design learning that specifically can develop mathematical literacy skills of mathematics pre-service teacher.

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